

Passive Stability and Active Control in a Rhythmic Task

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Wei K, Dijkstra TM, Sternad D. Passive stability and active control in a rhythmic task. *J Neurophysiol* 98: 2633–2646, 2007. First published September 19, 2007; doi:10.1152/jn.00742.2007. Rhythmically bouncing a ball with a racket is a task that affords passively stable solutions as demonstrated by stability analyses of a mathematical model of the task. Passive stability implies that no active control is needed as errors die out without requiring corrective actions. Empirical results from human performance demonstrated that actors indeed exploit this passive dynamics in steady-state performance, thereby reducing computational demands of the task. The present study investigated the response to perturbations of different magnitudes designed on the basis of the model's basin of attraction. Humans performed the task in a virtual reality set-up with a haptic interface. Relaxation times of the performance errors showed significantly faster returns than predicted from the purely passive model, indicative of active error corrections. Systematic adaptations in the racket trajectories were a monotonic function of the perturbation magnitudes, indicating that active control was applied in proportion to the perturbation. These results did not indicate any sensitivity to the boundary of stability. Yet the influence of passive dynamics was also seen: the pattern of relaxation times in the major performance variable ball height was consistent with qualitative predictions derived from the basin of attraction and racket accelerations at contact were generally negative signaling use of passive stability. These findings suggest that the fast return back to steady state was assisted by passive properties of the task. It was concluded that actors used a blend of active and passive control for all sizes of perturbations.

INTRODUCTION

Human actions necessarily express themselves as physical interactions with the environment where the biological system creates and is subject to forces that constrain the execution and the control of action. The stability of these forces is key to their control. Stability in postural and movement control is achieved by many different means: physiologically, it can arise from the stiffness of the muscular and tendinous tissues mediated by stretch reflexes; from a control perspective, stability is attained through feedback-based responses that correct for errors; a third source of stability is of mechanical and dynamical origin: the stability of the actor-environment system as it is set up by the task. This system may provide stability “for free” that is not brought about by the active compensations of errors, i.e., feedback control, but rather it is brought about by the inherent stability of the task. A prominent example that has demonstrated the importance of this source of stability is the passive dynamic walker. McGeer (1990) showed, by using both an analytic model and a multilink robot, that a purely mechanical

system without actuators and control could maintain a stable walking pattern when walking down a gentle slope.

Models of this type achieve dynamic stability without active control, relying solely on the passive dynamics of the physical constructs. Over the past two decades, a lot of attention and effort has been directed to extending the idea of passive dynamics to human-like biped robots walking on level ground and in 3D (Coleman and Ruina 1998; Collins et al. 2001). Several models demonstrated that walking in two dimensions has inherent stability, but to achieve walking in three dimensions, additional actuation was needed. However, the fundamental reliance on passive dynamics has offered these 3D walkers efficiency in energy expenditure, reduced demands in control, and elegant mimicry of human motion (Collins et al. 2005; Tedrake et al. 2004). The striking similarity of these minimally controlled walking machines with human walking suggests that passive dynamics may play an important role in shaping coordinated human behavior. Without downplaying the importance of muscular forces and their tuning by perceptual information in determining behavior, the passive dynamic walkers show that our understanding about control can be deepened by studying the motion that may emerge without control.

To tease apart the contributions of passive dynamics and active control, a motor task is wanted that first affords such a passively stable solution. Sternad and colleagues have shown this to be the case for the task of rhythmically bouncing a ball on a racket (de Rugy et al. 2003; Dijkstra et al. 2004; Schaal et al. 1996; Sternad et al. 2000, 2001). This task requires an actor to bounce the ball with a racket up in the air to a consistent height over repeated bounces. By viewing the racket as an oscillating planar surface and the ball as a point mass colliding with the racket with an inelastic impact, a simple mechanical model was derived for the ball-bouncing task. Stability analyses of this model yielded predictions about criteria for which this passive stability is achieved. Specifically, when the acceleration of the racket's upward movement is negative when contacting the ball, dynamic stability is indicated for the model.

Empirical studies of the ball-bouncing task confirmed that humans exploit the passive stability properties as predicted by the model. Experienced actors hit the ball with negative racket accelerations in a variety of experimental conditions: when different ball amplitudes were required (Schaal et al. 1996), when the movement of the ball was confined to the vertical dimension only (Sternad et al. 2000, 2001), or when the ball and the racket moved freely in three dimensions; when the ball was bounced by using a paddle moving downward instead of a

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hand-held racket moving upward to hit the ball (Schaal et al. 1996), or when the experiment was conducted in a virtual reality setup (de Rugy et al. 2003). In contrast, novice actors performed with positive impact acceleration and only gradually, within ~ 30 min of practice, tuned their movements to use negative impact acceleration (Dijkstra et al. 2004). This result was accompanied by decreasing variability supporting the interpretation that performance had improved with this change of strategy. This latter result highlighted that hitting the ball with negative impact acceleration was not an intuitive or trivial solution for actors. The task offers the advantage of stability, but it has to be learned. This was also shown by Siegler and colleagues in a study on learning new phase relations between ball and racket where subjects frequently performed the task with positive accelerations (Morice et al. 2007). In sum, the empirical results on the ball-bouncing task support the interpretation that actors exploit the stability properties of the task.

This ball bouncing model shares many features with the passive dynamic walking model. Both models are formulated over the actor-environment system with collisions as the primary form of interaction. Given that control of the continuous system is confined to intermittent moments of contact, it is a hybrid control system. Importantly, both tasks have multiple stable solutions with period-1 to period- n and chaotic solutions. The presence of multistability immediately raises the question about the boundary between these multiple solutions. How large can a perturbation be before the system ends up in another solution? How large is the basin of attraction? Results of passive dynamic walkers show that the basin of attraction is not very large, even with meticulous tuning of the parameters (Garcia et al. 1998; Schwab and Wisse 2001; Wisse and van Frankenhuyzen 2003). As a result, the bipedal robots are sensitive to initial conditions and demand a careful launch. These observations are in contrast to human walking, which is much more robust to perturbations and adaptive to different conditions pointing to the presence and importance of active control in human walking.

Although multistability similarly exists in the ball-bouncing map, only period-1 performance has been investigated thus far. Further understanding about the relationship between passive stability and active control can be obtained by examining the basin of attraction. Hence, the present study investigates the ball-bouncing task with periodic solutions under systematic perturbations that are designed in view of the basin of attraction for the period-1 solution. Will the actor discard the strategy of using passive stability and turn to active control with error feedback on a cycle-to-cycle basis? Alternatively, will the actor rely on passive stability without active error corrections when the ball is only slightly perturbed with perturbations inside the basin of attraction? Or will the actor adopt a mixture strategy with both the exploitation of passive stability and of perception-guided error corrections (Warren 2006)? In sum, are actors sensitive to the boundaries of the basin of attraction of the period-1 solution?

These questions can be answered by applying perturbations of different magnitudes. In a previous study, de Rugy et al. (2003) applied perturbations by randomly changing the coefficient of restitution of the ball-racket system on impacts leading to unexpected under- or overshooting of the ball with respect to the target height. Results showed that actors quickly reestablished negative acceleration at impact, indicating the

use of passive stability. Yet for all the applied perturbations, modulations of racket movements also indicated signs of active control. Specifically, the study found that directly following perturbation the periods of the racket cycles shortened or lengthened for smaller or larger ball amplitudes, respectively. The amplitudes of the racket movements remained largely unchanged. Although this study gave a first indication that actors "actively tracked passive stability," the sensitivity to the basin of attraction could not be tested because no theoretical analyses of the basin of attraction were available. Post hoc analyses revealed that the perturbations mostly took the system outside the basin of attraction. In fact, very small perturbations were excluded by design. To extend these first investigations, the current study provides a derivation of the basin of attraction for the period-1 attractor of the ball bouncing map. The experimental perturbations were designed such that they covered a wide range both inside and outside the basin. Furthermore, the coefficient of restitution of the racket, a critical variable influencing the shape of the basin of attraction, was also systematically varied. Based on the locations of perturbations in the basin, predictions about the response of a purely passive strategy can be made. These predictions are then compared with human performance to elucidate the relationship between passive dynamics and active control.

MODEL

The predictions are based on the same model that was derived in earlier work (Dijkstra et al. 2004; Guckenheimer and Holmes 1983; Holmes 1982; Sternad et al. 2001; Tuffilario et al. 1992). As mentioned in the preceding text, the task of periodically bouncing a ball with a racket to a target height affords a passively stable period-1 solution. This solution is entirely open-loop and, once initiated, requires no control or error correction. The period-1 solution co-exists with other solutions, in particular sticking solutions and period- n solutions. The latter solutions give rise to the period-doubling route to chaos and were an important motivation for the initial studies of the map (see Tuffilario et al. 1992). However, they are of no concern in the current context.

The ball bouncing map is based on the following three assumptions: 1) *Ballistic flight*: between the k th and the $k+1$ th bounce the vertical ball position $x_b(t)$ follows the ballistic flight equation

$$x_b(t) = x_b(t_k) + v_b^+(t - t_k) - (g/2)(t - t_k)^2 \quad t_k < t < t_{k+1} \quad (1)$$

with $x_b(t_k)$ the vertical ball position at the time of the last (k th) impact, v_b^+ , the ball velocity immediately after impact, and g the acceleration due to gravity (9.81 m/s^2). 2) *Instantaneous impact*: the impact is instantaneous such that the ball velocity immediately after impact v_b^+ is determined by

$$(v_b^+ - v_r) = -\alpha(v_b^- - v_r) \quad (2)$$

where v_b^- denotes ball velocity just before impact. v_r denotes the velocity of the racket at impact, and α denotes the coefficient of restitution, which captures the energy loss at the impact. The velocity of the racket does not change during impact because the mass of the racket is much larger than the mass of the ball. 3) *Sinusoidal racket movement*: the racket movement is a pure sinusoid

$$x(t) = a_r \sin \omega_r t \quad (3)$$

with amplitude a_r and angular frequency ω_r (Bapat et al. 1986).

The validity of these assumptions for bouncing a physical ball is discussed in Dijkstra et al. (2004) and Brody et al. (2002). Because the task is performed in a virtual set-up (see METHODS in the following text), the ballistic flight and the assumption of instantaneous impact are satisfied by design. The assumption of a pure sinusoid is not obeyed, not even in the virtual set-up: actors pick a periodic waveform that is slightly steeper than a sine wave in the ascending phase before the racket meets the ball. Unfortunately, there is no simple mathematical description of this waveform. However, we note that only position and velocity with which the racket hits the ball determine the ball trajectory. Thus for mathematical simplicity, we use an equivalent sinusoid that is close to the actual waveform at the impact: its equivalent frequency ω_r is calculated from the period between bounces and its equivalent amplitude a_r is calculated from the stationary phase of impact θ as (see Dijkstra et al. 2004, Eq. 8)

$$a_r = \left(\pi \frac{1 - \alpha}{1 + \alpha} \right) \frac{g}{\omega_r^2 \cos \theta} \quad (4)$$

From these assumptions, the ball-bouncing map can be derived as

$$v_{k+1} = (1 + \alpha) a_r \omega_r \cos \theta_{k+1} - \alpha v_k + (g/\omega_r)(\theta_{k+1} - \theta_k) \quad (5)$$

$$0 = a_r \omega_r^2 (\sin \theta_k - \sin \theta_{k+1}) + v_k \omega_r (\theta_{k+1} - \theta_k) - (g/2)(\theta_{k+1} - \theta_k)^2 \quad (6)$$

This is an implicit map with the two state variables v_k , the ball velocity just after impact, and θ_k , the racket phase of impact. The ball bouncing map has a period-1 attractor which is locally linearly stable when the acceleration at impact, denoted by AC, is bounded by

$$-2g \frac{1 + \alpha^2}{(1 + \alpha)^2} < AC < 0 \quad (7)$$

The period-1 attractor co-exists with other attractors: for AC more negative than the lower boundary, there exist attractors where the ball sticks to the racket for part of the cycle; for positive AC several attractors exist, among them the period-doubling route to chaos (Tuffilaro et al. 1992). It is important to stress that the constraint on AC in Eq. 7 is based on the assumption of stationarity. Moreover, stability also depends on other parameters such as amplitude or period; however, only when other parameters are kept constant. The only single parameter that concisely captures the possible bifurcations is the acceleration at impact AC. When comparing these predictions with experimental data, one needs to keep in mind that we only report trial means and not trial-to-trial data. Given the ubiquitous variability, it occasionally happened that positive racket accelerations are interspersed.

The domain of attraction of the period-1 attractor depends on the parameters of the map: g , the acceleration of gravity, α , the coefficient of restitution, ω_r , the racket frequency, and a_r , the racket amplitude. These parameters can be tightly controlled or determined in the experiment to obtain a good quantitative match with the model. The first two parameters, g and α , are independent of the actor and can be experimentally manipulated. Because the linear stability in the model and the domain of attraction depend strongly on α (see Fig. 1), this parameter was varied as an independent measure in the current study. ω_r

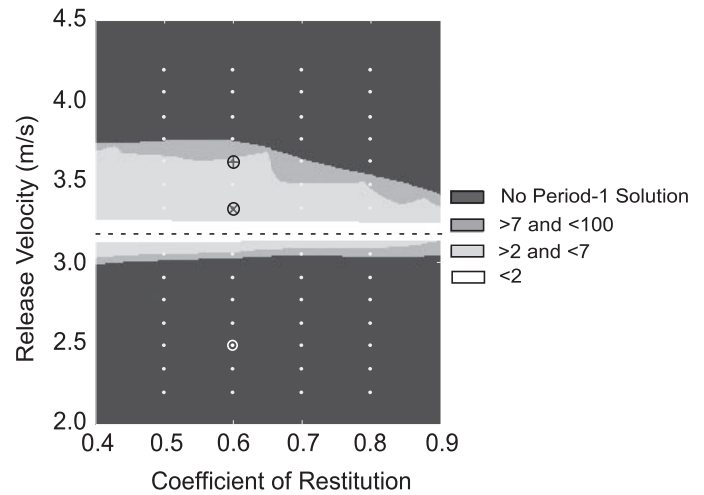


FIG. 1. Basin of attraction of the ball bouncing map. Dashed line, stationary release velocity. Gray shading, how many bounces or cycles are needed for the perturbations to die out. For example, the light gray means that the map took >2 and <7 bounces to return within a band of the stationary state (as defined in the text); the darker gray denotes that the map took any number of cycles between 7 and 100 to return to steady state. Black area, all those perturbations where the system assumes states other than period-1. White dots, perturbations used in the experiment: for each of the 4 coefficients of restitution, 14 different magnitudes of perturbations were applied. The 3 larger dots highlight 3 selected perturbations for $\alpha = 0.6$ for which the time course is illustrated in Fig. 2, following the ordering from the top to the bottom.

and a_r are more difficult to control experimentally because they do depend on the actors' performance. However, ω_r can be fixed by having actors bounce to a visual target. Because the ball amplitude determines the racket period (through the flight equation) and actors hit the ball at an approximately constant height relative to the floor, having a target at a fixed height relative to the floor fixes the racket period. The racket amplitude was not prescribed but the actual movement amplitude was estimated from the impact phase, using Eq. 4. The model parameter a_r was set accordingly for the calculations.

The domain of attraction of the period-1 solution was calculated by numerically iterating the ball bouncing map (Eqs. 5 and 6). The parameters and the initial conditions of the map were chosen to be as close as possible to the experiment. Gravity g was 9.81 m/s^2 and the racket frequency ω_r was $2\pi/0.65 \text{ rad/s}$, which is based on the grand average of the time between bounces over all actors and conditions of the present experiment. The grand average of impact phase, also determined from the subjects' data, varied with the coefficient of restitution α . For the experimentally used values of α , 0.5, 0.6, 0.7, and 0.8, the respective phase values were 6, 7, 8, and 13° . From these phase values, the equivalent racket amplitudes were calculated using Eq. 4 and entered into the calculations. For values of α between the experimentally used ones, spline interpolation and extrapolation was applied to obtain phase values. The initial velocity values were taken from the range 2–4.5 m/s (y axis in Fig. 1), and the initial phase values were set to the stationary phase for that particular coefficient of restitution. With these initial conditions and parameters, the ball bouncing map was iterated 100 times. To quantify the number of map iterations for the state to end up at the stationary state, a stopping criterion was necessary. In line with the observed variability in the control trials, a band of 0.1 m/s for velocity and 10° for phase around the stationary values was used. If the

state ended up inside the band within 100 iterations, these initial values were considered to be inside of the domain of attraction. Necessarily, the domain of attraction depended on the values chosen for the bandwidth. However, control simulations showed that the domain of attraction was relatively insensitive to the bandwidth.

The results of these computations are presented in Fig. 1, where the white and gray shaded areas denote the domain of attraction with increasing relaxation times (counted in number of cycles). The black area indicates initial conditions that did not converge to the stationary state within 100 iterations or that led to a sticking solution where the time between impacts was <1 ms.

The four vertical columns of dots in Fig. 1 indicate the different perturbation magnitudes that were used for the four different α values in the experiment. Figure 2 shows some simulation results for the three marked perturbation magnitudes in Fig. 1. *A* and *B* illustrate the time course of the two state variables at impact following a large perturbation that takes seven bounces to return to steady state. Equivalently, *C* and *D* show the state variables for a smaller perturbation that relaxes back within four bounces. The continuous time series on *E* and *F* illustrates how a sticking solution occurs. Note the perturbations were first designed based on preliminary calculations of the basin of attraction using parameter estimates from previous experiments. After having analyzed the data of the present experiment, the basin of attraction was recalculated as described in the preceding text to provide a better evaluation of the perturbations for the present data.

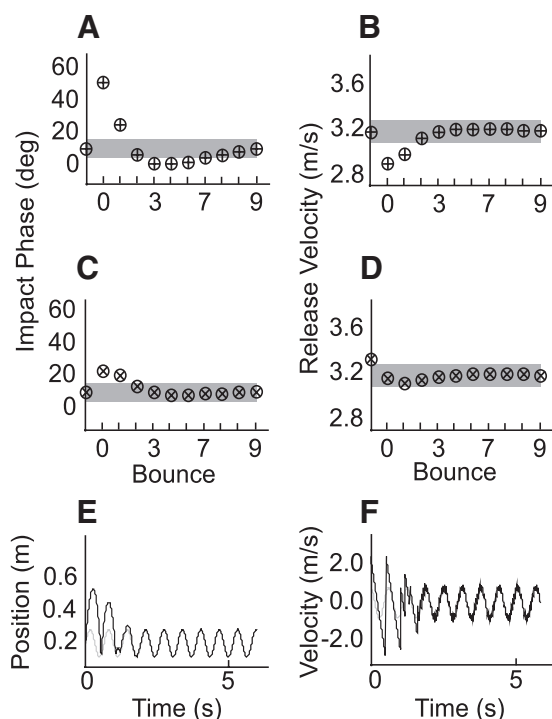


FIG. 2. Simulations for 3 selected perturbations of the ball bouncing map. *A* and *B*: time evolution of the state variables impact phase and ball release velocity, respectively. The shaded area denotes the bandwidth when the system is considered to be back at equilibrium. These 2 panels correspond to the upper perturbation in Fig. 1. *C* and *D*: equivalent plots for the middle perturbation in Fig. 1. *E* and *F*: time series of continuous position and velocity of both the racket and the ball, showing a sticking solution. This was obtained for the perturbation in the black area of Fig. 1.

From these analyses, the following predictions could be formulated.

PREDICTION 1. The basin of attraction has a boundary separating stable period-1 solutions from sticking and period- n solutions. Actors are sensitive to this boundary and rely on passive stability when perturbations are inside the basin of attraction. For perturbations outside the basin, two outcomes are conceivable: actors lose period-1 stability and switch to period- n (with $n > 1$) or sticking solutions or they adopt an active strategy that aims to correct for errors. In either case, a qualitative change in the behavior is expected. Note that sticking or period- n solutions have never been observed in the current experimental set-up (de Rugy et al. 2003). Hence active corrections are the more likely case that will be analyzed from modulations of the racket kinematics for destabilizing perturbations.

However, even if discontinuous changes in strategy across the boundary will not be observed, the basin of attraction still predicts qualitative changes in behavior as a function of perturbation magnitude and coefficient of restitution. Three qualitative predictions can be formulated.

PREDICTION 2A. With increasing perturbation magnitude, the time for returning to steady-state performance increases for all coefficients of restitution. Larger perturbations that take the system further out of the basin of attraction lead to longer relaxation times.

PREDICTION 2B. The relaxation time is longer for negative perturbations than for positive perturbations for all coefficients of restitution (i.e., for release velocities smaller than the average release velocity). This follows from the observation that the lower boundary of the basin of attraction is closer to the stationary state than the upper boundary.

PREDICTION 2C. For positive perturbations, the smaller coefficients of restitution have a wider basin of attraction. Hence for positive perturbations, the smaller coefficients of restitution should show faster returns than the higher coefficients of restitution. For negative perturbations, there should be no difference in relaxation time for different coefficients of restitution.

ALTERNATIVE PREDICTION 3. Actors apply active control for all errors regardless of their magnitude to ensure stable performance. Hence modulations of the racket trajectory should scale continuously with the magnitude of the perturbation. While in opposition to the previous results supporting the exploitation of passive stability, this statement also recognizes the fact that active control is needed to tune the system into this “passive dynamic” strategy.

METHODS

Participants

Seven volunteers participated, with ages ranging from 23 to 47 yr. With the exception of one participant, all others had some prior experience in performing this task. All participants reported to be right-handed and used their preferred right hand to bounce the ball with the racket. Before the experiment, all participants were informed about the procedure and signed the consent form approved by the Regulatory Committee of the Pennsylvania State University.

Experimental apparatus

In the virtual reality set-up, participants manipulated a real table tennis racket to bounce a virtual ball that was projected on a screen in front of them (Fig. 3A). Participants stood ~ 0.5 m behind a back-projection screen with width 2.5 m and height of 1.8 m. A PC (2.4 GHz Pentium CPU, Windows XP) controlled the experiment and generated the visual stimuli with a graphics card (Radeon 9700, ATI). The same PC also acquired the data using a 16 bit A/D card (DT322, DataTranslation). The images were projected by a Toshiba TLP 680 TFT-LCD projector and consisted of $1,024 \times 768$ pixels with a 60-Hz refresh rate. Accelerations of the racket were measured using a solid-state piezoresistive accelerometer mounted on top of the racket (T45-10, Coulbourn). The mechanical brake acted on the rod that was attached to the racket and was controlled by a solenoid (Magnet-Schultz type R 16×16 DC pull, subtype S-07447). A light rigid rod with three hinge joints was attached to the racket surface and ran through a wheel whose rotation was registered by an optical encoder (Fig. 3B). Its accuracy was one pulse for 0.27 mm of racket movement. The pulses from the optical encoder were counted by an onboard counter (DT322). The racket could move and tilt with minimal friction in three dimensions, but only the vertical displacement was measured. Images of racket and ball position were shown on-line on the back projection screen using custom-made software. The maximal velocity of the ball was estimated to be ~ 30 pixels/s on average. However, around the apex of the ball trajectory, where participants tend to focus their attention, the ball velocity is an order of magnitude smaller. The delay between real and virtual racket movement was measured in a separate experiment and found to be 22 ± 0.5 (SD) ms on average.

The virtual racket was displayed at the same height from the floor as the real racket, and its displacement was the same as that of the real racket. The movement of the ball displayed on the screen was

governed by ballistic flight and an instantaneous impact event when the virtual racket impacted the virtual ball. Just before the virtual ball hit the virtual racket a trigger signal was sent out to the mechanical brake that was attached to the rod. The trigger signal was sent out 15 ms before the ball-racket contact to overcome the mechanical and electronic delay of the brake. The brake applied a brief braking force pulse to the rod to create the feeling of a real ball hitting the racket. The duration of the force pulse (30 ms) was consistent with the impact duration observed in a real ball-racket experiment (Katsumata et al. 2003). The brake force was not scaled to the relative velocity of the ball and the racket but stayed the same for all impacts. The equipment did not permit to measure the actually applied force.

The computer program controlling the experiment would read the latest racket position from the optical encoder and racket acceleration from the accelerometer. When the racket was away from the ball, the program would update the ball positions based on the ballistic flight equation. On the 2.4-GHz computer under Windows XP this led to an update rate of ~ 800 Hz. When the ball and racket were close, the computer program would keep a running estimate of time to contact and control the brake accordingly. The increased computational load led to a slow-down of the update rate to ~ 250 Hz in the 30 ms surrounding an impact. The update rate was not fixed because Windows XP is not a real-time operating system and thus timing is not deterministic. Hence all data were time-stamped using the high-resolution timer on the Pentium CPU with an accuracy better than 1 ms.

Procedure and experimental conditions

Prior to each experiment, the participant was placed on a support base to adjust for height differences. The support height was adjusted such that the height of the hand-held racket, when held with the forearm horizontally, was 10 cm above its lowest position. Each trial

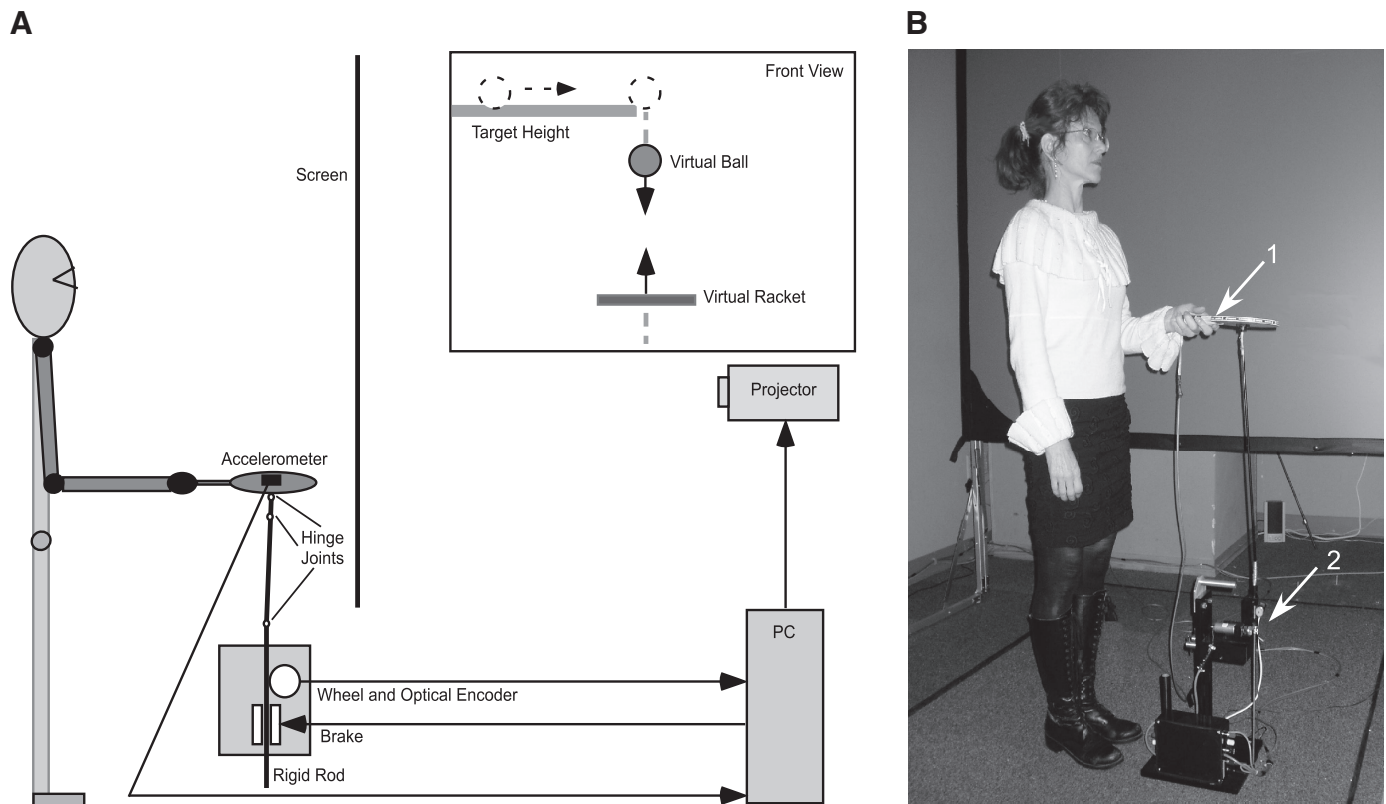


FIG. 3. A: sketch of the virtual reality setup for the ball bouncing task, a side view and a front view of the screen display. B: photographic view of the hardware used for the acquisition of the racket position and acceleration and the brake. Arrow 1, accelerometer mounted on the racket; arrow 2, brake and the optical encoder mounted on the hardware.

began with a ball appearing at the left side of the screen and rolling on a horizontal line extending to the center of the screen (see Fig. 3A, inset). On reaching the center, the ball dropped from the horizontal line (0.7 m high). The task instruction was to rhythmically bounce the ball for the duration of a trial (55 s) as accurately as possible to the target line (the same line that the ball started on). The experiment consisted of a total of 80 trials, which were collected in two sessions. Each session lasted ~1 h.

The entire experiment was divided into four blocks of 20 trials, one block for each value of the coefficient of restitution α : 0.5, 0.6, 0.7, and 0.8. The blocks were presented in either ascending or descending order, counterbalanced among participants. The first two and the last two trials of each block were control trials without any perturbation. In the remaining 16 experimental trials, perturbations were applied at random impact times. A perturbation was created by an abrupt change of the ball release velocity immediately after the ball-racket impact. This led to an unexpected ball amplitude without any other noticeable change in the ball trajectory. This perturbation in velocity was chosen randomly from 14 magnitudes which were added or subtracted from the current release velocity $-1.0, -0.86, -0.71, -0.57, -0.43, -0.29, -0.14, 0.14, 0.29, 0.43, 0.57, 0.71, 0.86$, and 1.0 m/s (see Fig. 1). Negative values led to smaller ball amplitudes, positive values to larger ball amplitudes. With an average ball amplitude of 0.55 m and a corresponding average bounce period of 650 ms, the effect of these perturbations can be converted to deviations from the target height. The largest positive perturbation caused an overshoot of 0.37 m above the target, and the largest negative perturbation an undershoot of 0.27 m. The smallest perturbation of 0.14 m/s caused an overshoot of 0.047 m and -0.14 m/s an undershoot of 0.045 m. The 14 different magnitudes of perturbations ranging from -1 to $+1$ m/s were labeled as $P-7, P-6, P-5, \dots$, to $P+6, P+7$.

The complete set of 14 perturbations for each α was delivered on two successive trials with 7 perturbations within one trial in randomized order. As each trial had ~80–90 bounces, the perturbations occurred randomly on the 8th, 9th, or 10th bounce relative to the previous perturbation. Across the 16 experimental trials, the set of 14 perturbations could be administered eight times. With randomization of both time and magnitude of perturbation, participants were unable to anticipate the ball amplitude or the time of the perturbations.

Data reduction and analysis

The raw data of the racket displacement and acceleration were resampled at a fixed frequency of 500 Hz and filtered with a fourth-order Savitzky-Golay filter with a window size of 0.01 s on both sides (Gander and Hrebíček 2004). The filter order and window size were chosen empirically to remove measurement noise while not excessively smoothing the signals. The Savitzky-Golay filter is superior for smoothing data that have abrupt changes as compared with conventional filters like Butterworth filters. These abrupt changes occurred in the data when the racket exhibited a sudden drop in acceleration caused by the brake. The ball displacement was generated by the computer, so it contained no measurement noise. Therefore no filtering was necessary. As a verification of our filtering procedure, the racket displacement was double-differentiated using a Savitzky-Golay filter and compared with the acceleration data collected by the accelerometer. Figure 4A illustrates that the two signals showed a good match, supporting the validity of the data acquisition. The time series shows two cycles with two ball-racket impacts and indicates the moment of impact as estimated from the double-differentiated position signal and the raw accelerometer signal. On average, the accelerometer signal rendered estimates that were 0.52 m/s^2 more positive than the position signal. To be conservative and given that the variability of the impact accelerations from the raw acceleration signal was smaller, we opted to report the estimates from the raw acceleration data.

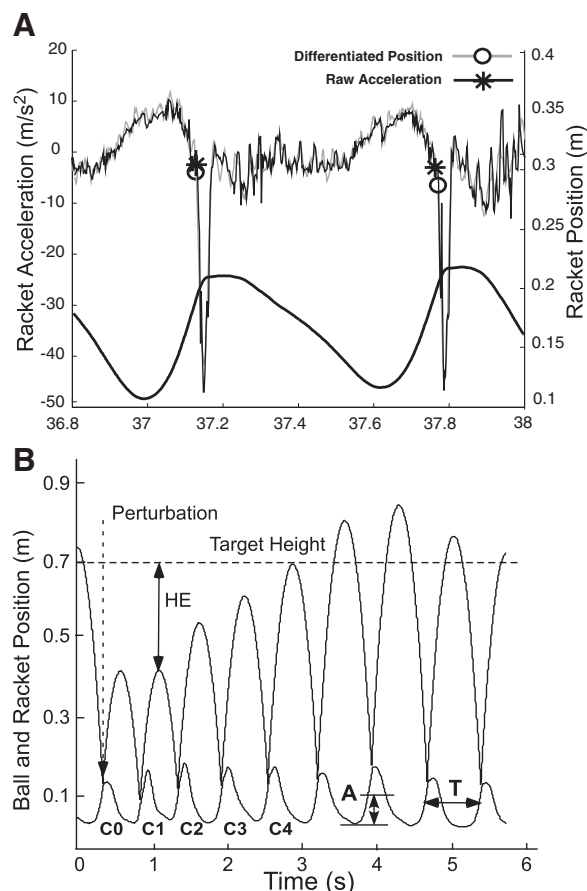


FIG. 4. A: segments of time series of the double-differentiated position signal and the raw accelerometer signal together with the racket trajectory to illustrate their match. B: segment of an exemplary trial with a large perturbation ($P-7$). The cycles following the perturbed impact are marked as C0, C1, C2, etc. For the definitions of the dependent measures, see text for details.

Dependent measures

Figure 4B illustrates the primary dependent measures. Performance was evaluated by the ball height error, HE, which was defined as the signed difference between the maximum ball height and the target height. Height error was equivalent to the state variable ball velocity at the moment of release from the racket, as this velocity determined the subsequent ball amplitude in the gravitational field if the impact position was relatively constant. The racket amplitude, A, was calculated as half the distance between the minimum to the maximum of the racket trajectory during one cycle. The racket period, T, was calculated from the intervals between the times of peak velocities of successive bounces. The acceleration of the racket at impact, AC, was determined from the accelerometer signal one sample before the time of impact.

Relaxation time after perturbation

To quantify the return to stationary bouncing after a perturbation, we fitted the return of the height error over cycles with an exponential function. The cycles were labeled in sequential order starting from the perturbed cycle directly following the perturbed impact, labeled C0; the following cycles were labeled C1, C2, and so on (Fig. 4B). To estimate the relaxation time after the perturbation at C0, a Levenberg-Marquardt least-squares fitting of an exponential function was performed using the following functional form (Matlab 6.5, Mathworks)

$$y_k = y_0 e^{-k/\tau} + y_\infty \quad k \in [0 \dots 5] \quad (8)$$

where y_k was the dependent measure and k was the cycle number with $k = 0$ denoting the perturbed cycle, C0. Fitted parameters were the amplitude of the perturbation y_0 , the final level y_∞ and the relaxation time τ . We fitted the grand mean over eight repetitions and seven participants of each dependent measure as a function of cycle for each of the four α conditions. Although the exponential fits were satisfactory in general, the smallest perturbations $P-1$ and $P+1$ could not be reliably fitted as the return took effect within one cycle. For $P-2$, $P+2$, and $P-3$, a portion of the fits was acceptable and was included in the figures and further analyses. The reported means were calculated over the values that were reliably obtained.

Exit times

To calculate whether and when the racket trajectory was actively modified due to the perturbations, the racket cycle directly following the perturbation was compared with the unperturbed racket trajectories during the control trials. The time series of racket position of the two control trials at the start of each α block (total ~ 170 cycles) was parsed into cycles at the moment of impact, then overlaid and averaged. The median and interquartile range were calculated to render a representative control cycle with an error band for comparison. The exit times were defined as the time from the application of the perturbation at impact to the time when the median racket trajectory exited the error band (1.5 times the interquartile range).

RESULTS

Performance during control trials

RACKET ACCELERATION. Given that the objective of the study examined perturbations away from stable performance the data first had to be examined whether actors indeed performed the task in accord with the criteria of passive stability. Hence, performance was evaluated in the control trials (4 trials for each of the 4 α condition). The primary measure indicating performance at passive stability is the mean impact accelerations AC across all bounces of one trial (typically 70–80 bounces during 55-s-long trials). The mean values across the four control trials, determined separately for each of the four α conditions, are listed for all seven participants in Table 1. Overall, participants showed negative AC values as predicted by the model and seen in previous studies, with only two exceptions: *participants 1* and *2* had small positive values for $\alpha = 0.8$ and 0.5 , respectively. Excluding these two cases, these results verified that all participants indeed performed the task consistent with criteria for passive stability. A 4 (α) $\times$ 7 (participant) ANOVA was performed on these data with participant treated as a random factor. The results showed no significant differences between different α values, $F(3,84) = 0.63$, $P = 0.607$. The main effect of participant and the

interaction were significant, $F(6,84) = 9.98$, $P < 0.0001$, and $F(18,84) = 3.02$, $P < 0.0001$, respectively. The grand mean of these control data (-1.75m/s^2) was used as the baseline for the design of the perturbation magnitudes and the calculations of the basin of attraction.

HEIGHT ERROR. Performance during control trials was also evaluated in terms of the primary performance measure, height error HE. Subjecting the mean values determined across each trial in the four α conditions to a 4 (α) $\times$ 7 (participant) ANOVA did not identify significant difference between α conditions. Differences between individuals were significant, $F(6,84) = 7.90$, $P < 0.0001$. Overall, actors tend to slightly overshoot the target by an average of 0.016 , 0.017 , 0.017 , and 0.023 m for $\alpha = 0.5$, 0.6 , 0.7 , and 0.8 , respectively.

RACKET PERIOD AND AMPLITUDE. For a better characterization of the task performance, the continuous racket trajectories during steady state were assessed by their mean period and amplitudes per trial. The same 4 (α) $\times$ 7 (participant) ANOVA performed on period yielded significant differences between different α values, $F(3,84) = 8.91$, $P < 0.005$, and between different participants, $F(6,84) = 20.56$, $P < 0.0001$. The interaction between participants and α was also significant, $F(18,84) = 4.44$, $P < 0.0001$; the racket periods tended to increase for larger α conditions (634 ± 38 , 642 ± 44 , 654 ± 45 , 679 ± 44 ms). One cause for this trend was that in the higher α conditions participants impacted the ball at slightly lower positions but this observation was not statistically significant.

The equivalent ANOVA on amplitudes revealed a decreasing trend for higher α , $F(6,84) = 173.22$, $P < 0.0001$, indicating that subjects moved the racket less when the ball-racket contact was bouncier: 0.067 ± 0.007 , 0.049 ± 0.005 , 0.036 ± 0.003 , and 0.025 ± 0.005 m for α conditions 0.5 , 0.6 , 0.7 , and 0.8 , respectively. The main effect for participant and the interaction were significant, $F(6,84) = 3.87$, $P < 0.05$, $F(18,84) = 6.76$, $P < 0.0001$.

Performance after perturbations

HEIGHT ERROR. We first report the perturbations of the height error as the task demanded a minimization of height error. Figure 5 displays the grand averages of HE over all repetitions and all seven participants as a function of cycle number directly before and after the perturbation. The error bars were calculated as the average over all individuals' SDs across trials. Due to space limitations, only eight perturbation magnitudes are displayed. As to be expected, large effects of the perturba-

TABLE 1. Impact accelerations (m/s^2) of individual participants in control trials

	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	Participant Mean
Participant 1	-1.25 ± 3.29	-0.07 ± 2.11	-2.26 ± 2.10	0.29 ± 2.04	-0.82 ± 2.39
Participant 2	1.00 ± 2.70	-0.11 ± 2.68	-1.08 ± 2.75	-0.72 ± 2.09	-0.23 ± 2.56
Participant 3	-2.43 ± 2.28	-2.22 ± 1.98	-1.43 ± 1.50	-1.50 ± 1.25	-1.90 ± 1.75
Participant 4	-4.21 ± 2.48	-3.56 ± 1.74	-4.37 ± 1.67	-3.64 ± 1.09	-3.95 ± 1.75
Participant 5	-3.21 ± 2.39	-3.46 ± 1.79	-2.66 ± 2.05	-2.76 ± 1.56	-3.02 ± 1.95
Participant 6	-0.59 ± 2.13	-0.34 ± 1.94	-1.60 ± 1.90	-2.57 ± 2.05	-1.28 ± 2.01
Participant 7	-0.96 ± 2.46	-0.68 ± 2.04	-0.84 ± 1.13	-2.00 ± 0.91	-1.12 ± 1.64
Grand mean	-1.66	-1.49	-2.03	-1.84	-1.75

The values are means $\pm$ SD across 4 trials.

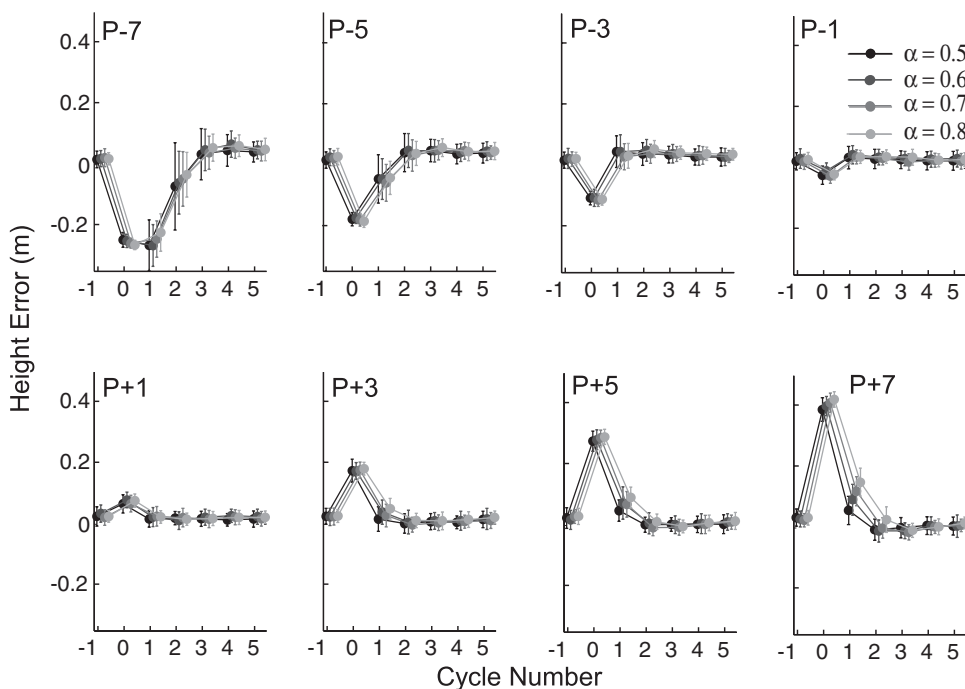


FIG. 5. Grand averages over all repetitions and participants of height error (HE) plotted as a function of cycle number. The 8 panels show the data for 8 perturbation magnitudes. Different α conditions are shown by different gray shades. The error bars denote the average of the individuals' SDs across trials. The data points for the 4 α conditions are slightly shifted for better visibility.

tions were observed at the perturbed cycle C0 and HE deviated from the baseline level with a magnitude that scaled with the perturbation; larger perturbations lead to larger HE values, indicating that the experimental perturbation had a significant effect on the performance. During subsequent cycles, HE showed an approximately exponential return back to pre-perturbation values. This return was not symmetrical for positive and negative perturbations of the same magnitudes. Comparing the largest negative perturbation, $P-7$, with the largest positive perturbation, $P+7$, it was apparent that $P+7$ showed a faster return, even though for $P+7$, the ball amplitude deviated more from target (approximately $+0.37$ vs. -0.27 m for $P+7$ vs. $P-7$, respectively).

The second observation is that for the small perturbations $P+1/P-1$, $P+2/P-2$ and also $P-3$ the time of return to pre-perturbation values was as short as one cycle and only approximately three cycles for larger perturbations. Even the larger perturbations that took the system outside of the basin of attraction were compensated mostly within three cycles. This indicates that participants managed to recover from perturbations faster than the model predicted. There was no visible difference between the different coefficients of restitution for negative perturbation; in contrast, for large positive perturbations, higher values of α had slower relaxation times. These observations will be quantified by the following analyses.

RELAXATION TIMES OF HEIGHT ERROR. To quantify the rate of return τ , an exponential function was fitted to the HE data (Eq. 8). Although HE is not a state variable, it is proportional to the square of the ball velocity after impact, which is a state variable. The exponential curve fits are illustrated for 10 of the 14 perturbation magnitudes for $\alpha = 0.6$ in Fig. 6. The two smallest perturbations $P+1/P-1$ and $P+2/P-2$ could not be fitted. The different lines represent the fitted curves for the grand average of HE over all eight repetitions and seven actors and shown over eight cycles after the perturbation (from C0 to C7). We chose to fit the grand average rather than per-trial or

per-subject fits because the resulting relaxations were quite variable. The curve fits rank-order with perturbation magnitude, with larger perturbations producing larger perturbation amplitudes (y_0) and longer relaxation times, τ . The R^2 values for the 10 fits for all participants ranged between 0.84 and 0.99, with an average of 0.96.

The τ estimates obtained from the fits are plotted in Fig. 7. The τ values of the very short returns are overlaid by the gray box to indicate that these values have to be interpreted with caution. We also distinguished between perturbations inside

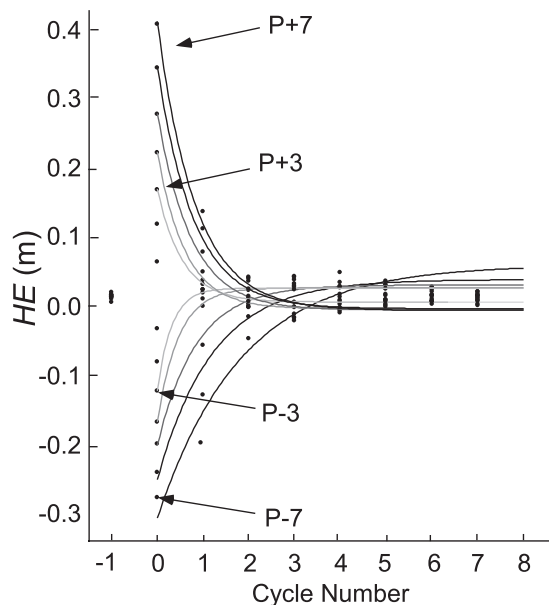


FIG. 6. Exponential curve fits of the height error HE for 10 of the 14 different perturbation magnitudes for a coefficient of restitution of 0.6. The dots are grand averages over repetitions and participants. The height errors for $P+1/P-1$ and $P+2/P-2$ are not fitted with exponential curves.

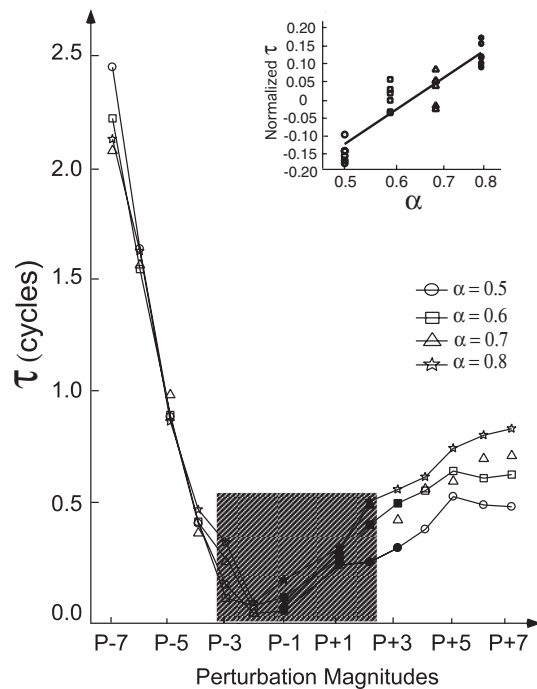


FIG. 7. Relaxation times τ obtained from the exponential fits of (HE). Different α conditions are shown by different lines and symbols. The gray shaded box indicates that the τ values for the smallest perturbations have to be interpreted with caution (see text for discussion).

and outside the basin of attraction, with hollow symbols for perturbations outside and filled symbols for inside the basin of attraction. Reliable relaxation times were almost only confined to perturbations outside the basin of attraction. This figure provides a first basis to test the predictions. Prediction 1 anticipated a qualitative change in behavior from perturbations inside to outside the basin of attraction. However, the figure does not reveal such discontinuous change in the relaxation constants. Still, the τ values show a distinct pattern that can be

evaluated in view of the second set of predictions. Consistent with prediction 2a, relaxation times were higher for larger perturbations. Further, there was a noticeable asymmetry between negative and positive perturbations; relaxation times were larger for large negative perturbations than for their corresponding positive ones, consistent with prediction 2b. It can also be seen that different α conditions did not induce differences in relaxation times across all perturbation magnitudes with the only exception that for positive perturbations lower α conditions appeared to show shorter relaxation times. This finding supports prediction 2c: the basin of attraction narrows for larger α , but only on the positive side. To corroborate this impression, the τ values of the positive perturbations P+3 to P+7 were regressed against the four α values (see Fig. 7, inset). Note the τ means of each perturbation magnitudes were subtracted for better comparison. The regression has a R^2 of 0.84 supporting the significant changes of τ with α .

RACKET ACCELERATION. Fig. 8 shows the grand averages over repetitions and participants of AC for eight perturbations plotted over cycles before and after the perturbations; as for height errors, the error bars are the average over the individuals' SDs across trials. Only the three largest negative perturbations (P-7, P-6, P-5) introduced significant deviations in AC from the baseline. For all positive perturbations and small negative perturbations, AC did not change. This observation was supported by calculating the percentage of trials that had a significant change in the cycle directly after a perturbation. In the first cycle after a perturbation (C1), the percentages of trials with a change of >1 SD from mean control trial performance (baseline) were 10, 7, 4, 9, 10, 11, 8, 12, and 11% for perturbation magnitudes P-2 to P+7, respectively. (Note that at cycle C0 at which the perturbation was applied, 6% of trials had AC values different from baseline.) Only for the three largest negative perturbations (P-7, P-6, P-5), a high percentage of trials showed deviations from baseline: 83, 78, and 59% of trials showed changes in C1 >1 SD. The values of AC

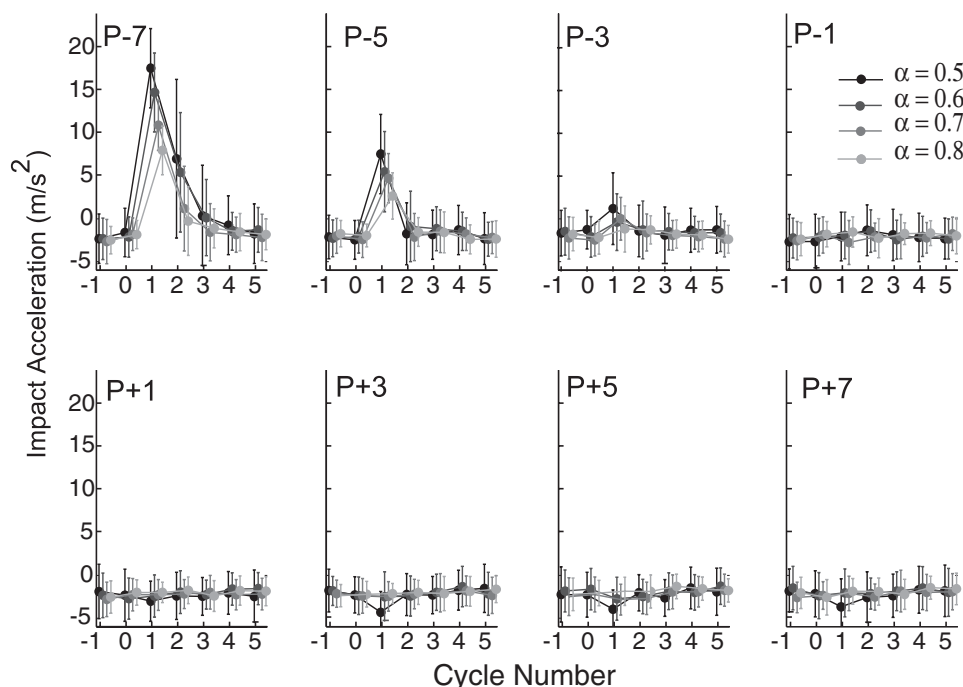


FIG. 8. Impact acceleration AC (averages over participant and repetition) plotted as a function of cycle number. The error bars denote the average of the individuals' SDs across trials. Each of the 8 panels shows the data for 1 perturbation magnitude. Different α conditions are shown by different lines. The data points for the 4 α conditions are slightly shifted for better visibility.

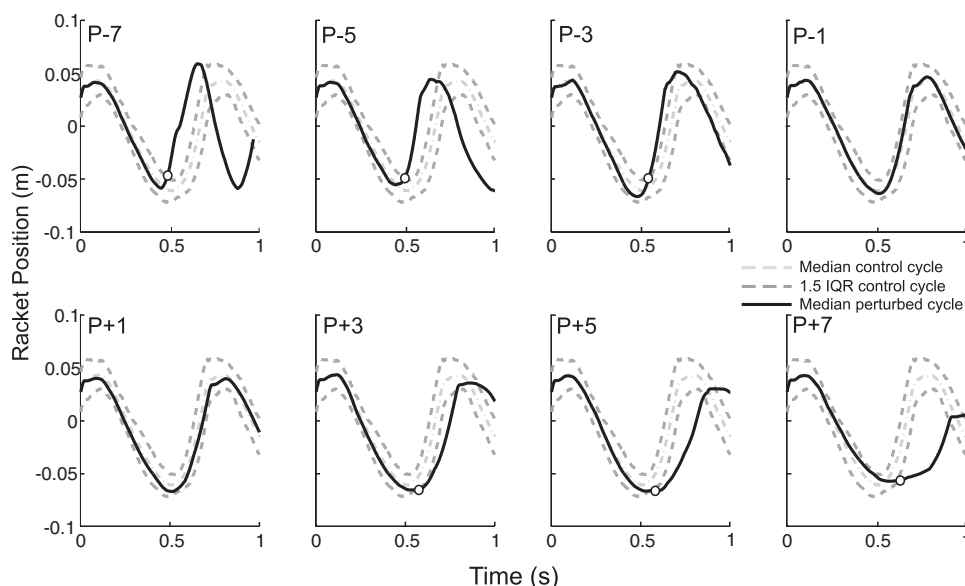


FIG. 9. Exemplary trajectories from 1 participant showing the adaptations of the racket trajectories to the perturbed ball trajectory in the cycle directly after the perturbation. For selected perturbations, the median perturbed racket trajectory (calculated over 8 trials for the same perturbation condition) is shown by the solid black line. The dashed gray lines are the median and 1.5 of the interquartile range calculated from the control trials of the same subject. The white circles denote the moment where the median trajectory exits the error band of the control trials. The beginning of the trajectory segment is at the perturbed impact, the interval is 1 s.

jumped from around -2.0 m/s^2 at the preperturbation cycles to around $+10 \text{ m/s}^2$ in *C1*. The large positive impact acceleration suggested that the ball contacted the racket earlier in the upward movement due to the undershoot of the ball amplitude caused by these three large negative perturbations. In sum, the perturbations had minimal effects on the racket-ball contact and the racket continued to contact the ball with negative accelerations even for large perturbations outside the basin of attraction.

Thus far, the data give some seemingly inconsistent answers to the predictions. 1) The return behavior is quantitatively inconsistent with the predictions resulting from passively stable behavior, as the return is significantly faster. 2) There is no evidence for a qualitative change when the perturbations are inside or outside the basin of attraction. 3) There are several qualitative features that speak to the fact that the basin of attraction does play a role in the return behavior. 4) The racket contact results (AC) indicate that participants overall continue to use passive stability to recover from perturbations. These results point to the fact that actors must have made adjustments in their racket trajectories to achieve such fast recovery from the perturbations. If so, were these adjustments in the racket movements sensitive to the boundary of the basin of attraction, relying on passive stability for small perturbations? These questions were addressed in a detailed analysis of the racket kinematics.

ACTIVE MODULATION OF RACKET TRAJECTORIES. A first assessment of the strategy that actors applied to deal with perturbations can be obtained from Fig. 9, which shows illustrative racket trajectories of one participant. For the selected eight perturbations, the median of the racket cycles directly after the perturbation is shown for 1 s (going slightly beyond the impact). For comparison, the median trajectory and its interquartile range from two control trials were calculated. As can be seen, the racket trajectory was modified depending on the magnitude of the perturbation. For *P-7* the racket cycle shortened such that the racket trajectory exited the error band after approximately half a cycle as indicated by the white dot in Fig. 8. Analogously, when the positive perturbations became larger, the racket cycle immediately “stretched” and exited the

error band after the valley. With smaller perturbations, the exit times became longer and in this participant no deviations were observed for *P-1* and *P+1*. (For this participant, the conditions *P-2* and *P+2* induced modulations with exit times). Over all participants, the smallest perturbations *P-1* and *P+1* lead to deviations in only 24% of the trials. For *P-2* and *P+2*, 80% of the trials showed significant deviations measured by exit times. For all larger perturbations, every trial showed exit times.

When these exit times were calculated and pooled for all participants, a clear pattern became visible (Fig. 10): For both negative and positive perturbations, there was a monotonic increase from larger to smaller perturbations. As there was no difference for the four α conditions, the values were pooled and plotted against perturbation magnitude. The missing exit times for the smaller perturbations were unsystematically distributed across participants and α conditions. The means were calculated across the existing number of data points and missing values were treated as missing cells. The less systematic observation for the smallest perturbations *P+1* and *P-1* are reflected in the large error bars. The monotonic scaling for positive and negative perturbations was supported by two

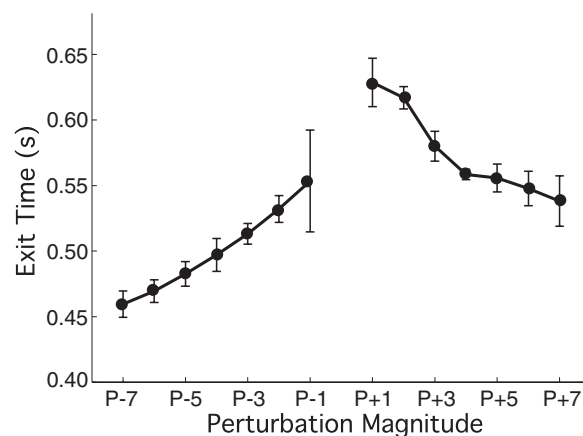


FIG. 10. Grand average of exit times for the perturbation magnitudes averaged over the α conditions. The error bars denote the SDs.

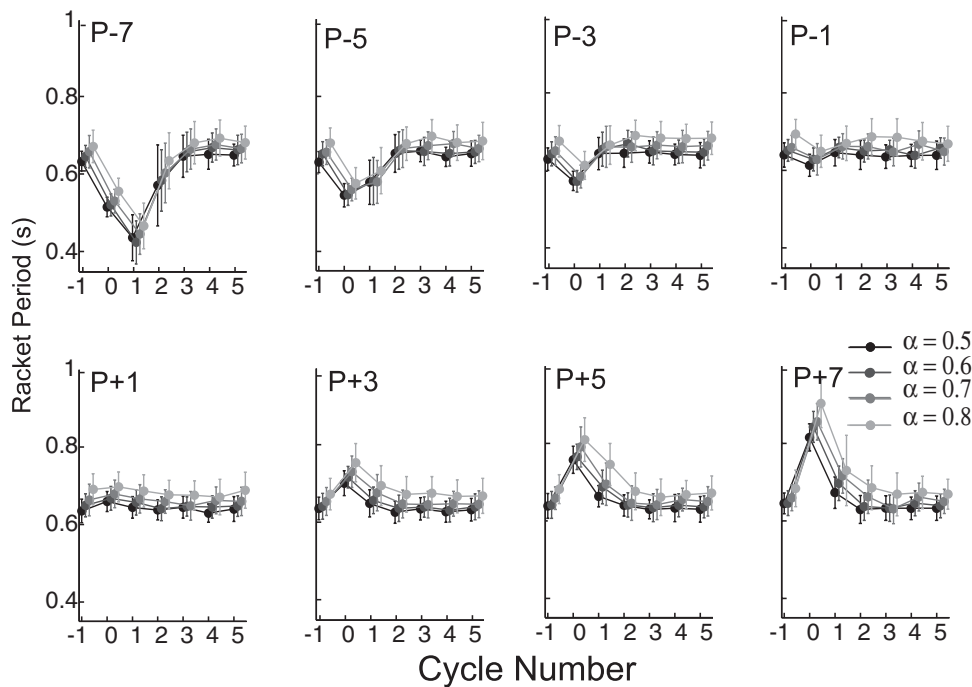


FIG. 11. Period averages (over participant and repetition) between impacts T as a function of cycle number. The error bars denote the average of the individuals' SDs across trials. Each of the 8 panels shows the data for 1 perturbation magnitude. Different α conditions are shown by different lines.

separate highly significant linear regressions, $R^2 = 0.90$ and 0.98 , $P < 0.001$. The exit times for positive perturbations were on average longer by 73 ms. With a view to predictions 1 and 3, this figure shows a clear continuity in the response to perturbations without any qualitative changes across the boundary of stability. With exception of $P+1$ and $P-1$, racket modulations are always present.

RACKET PERIODS AND AMPLITUDES. Figures 11 and 12 illustrate how the racket periods T and amplitudes A changed in the face of perturbations. As for AC and HE, T and A were averaged across trials and participants of each α condition and plotted against cycle number for different perturbation magnitudes.

The period results in Fig. 11 show a rank ordering of α conditions: smaller α conditions lead to shorter racket periods, similar to the behavior observed in the control trials. Additionally, T also showed systematic deviations from baseline in C0 and the magnitudes of deviations scaled with the perturbation magnitude similar to HE. The changing pattern of T indicates that the racket periods were adjusted according to the perturbed ball trajectory such that the racket periods were scaled with the ball amplitude after the perturbations. This coupling between the racket and the ball was in effect during the very first cycle after perturbation as the exit times also revealed. There was no discernable difference between α conditions.

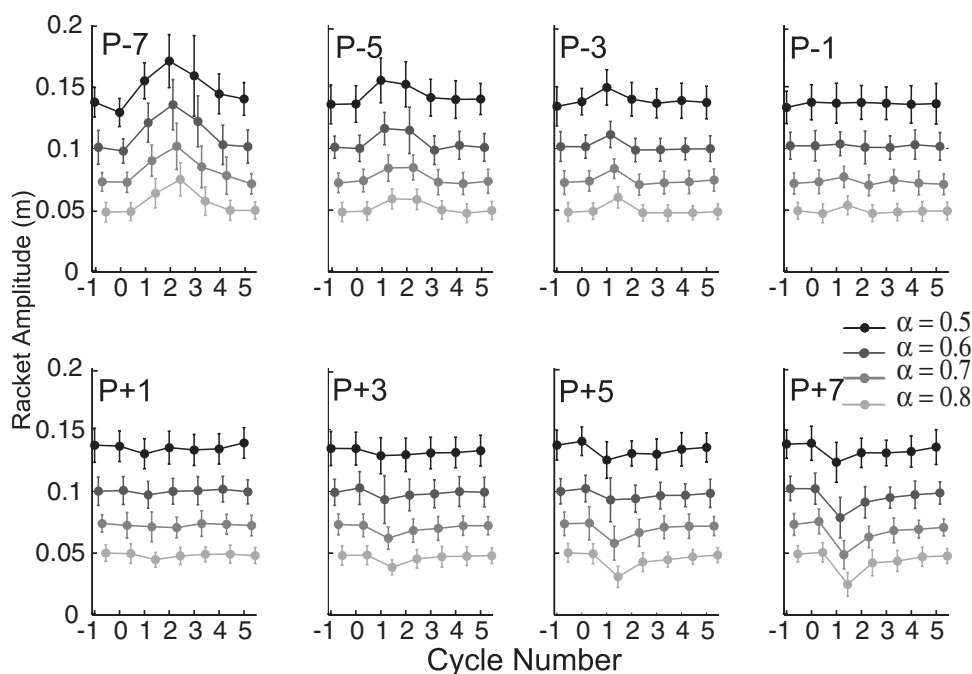


FIG. 12. Racket amplitude averages A (over participant and repetition) plotted as a function of cycle number. The error bars denote the average of the individuals' SDs across trials. Each of the 8 panels shows the data for 1 perturbation magnitude. Different α conditions are shown by different lines.

Figure 12 summarizes the amplitudes A : a strong dependence on α , i.e., the more elastic the ball-racket contact was (higher α value), the smaller was A . Note that there was no change at C0 because amplitude A was defined as the half-distance between minimum and maximum and enclosed the perturbed impact and did not allow sufficient time to show any perturbation effect. In contrast, period T comprised a longer interval after the perturbation such that the perturbation effect was clearly seen in C0. However, A systematically changed in C1, increasing or decreasing depending on the sign of the perturbation. The larger the perturbation, the larger were the changes in A . However, for the very small perturbations, the changes in A were relatively small. There was no discernable difference between α conditions in terms of changing pattern of A .

DISCUSSION

The present experiment examined the role of passive stability and active control in a rhythmic perceptual-motor task where participants aimed to steadily bounce a ball to a given target height. A series of previous studies on steady-state performance of the same experimental task established that actors are sensitive to and exploit the stability properties defined by the dynamics of the racket and ball movements (Dijkstra et al. 2004; Schaal et al. 1996; Sternad et al. 2000, 2001). This strategy implies that small errors, such as unintended deviations of the ball from the target height, require no corrections as the ball trajectory passively relaxes back to its steady state. Large perturbations, however, may take the system outside its basin of attraction and stable period-1 solutions are lost if the errors are not actively corrected for. In extension of these studies, Siegler and colleagues have shown in the same virtual task as in this study that novice actors can also perform the task with positive accelerations, a strategy that appears less efficient but also leads to continued successful performance of the task (Morice et al. 2007). These results may be taken as an indication that active control was applied. The present study presented perturbations to examine whether actors changed their strategy to correct for such induced errors and whether their strategy is dependent on the magnitude of perturbations.

A set of 14 perturbations of different magnitudes was designed for each of the four coefficients of restitution based on the basin of attraction that was determined from the ball bouncing model. As the stability boundaries were different for different coefficients of restitution α , the experiment was conducted with four α values. The predictions about relaxation behaviors after perturbations were based solely on the passive dynamics of the ball-racket system as the ball bouncing model does not include any active control of the racket movements. The first prediction was that if actors exploit this passive stability and do not change their racket movements according to perceived error information as long as errors are small, a sharp change in behavior should be seen for perturbation inside and outside the basin of attraction; for larger perturbations, actors either lose period-1 stability or apply active corrections to their racket movements to regain stability. Such sensitivity to the boundary of stability is present in purely passive dynamical walking models where disturbances outside the narrow basin of attraction make the bipeds fall (Garcia et al. 1998; Schwab and Wisse 2001). A second set of predictions formu-

lated more qualitative expectations. *a)* With increasing perturbation magnitude, the time for returning to baseline performance increases for all α . *b)* The relaxation time is longer for negative perturbations than for positive perturbations for all α . And *c)* for positive perturbations, the smaller α should show faster returns than the higher α . The alternative hypothesis was that actors perceived all applied errors, even small errors, and actively adjusted their racket movements to regain steady state performance. This third prediction adopts the extreme position that actors always correct for errors in their performance and compensatory actions should continuously scale with the magnitude of the perturbation.

The previous study by de Rugy et al. (2003) already examined perturbations in the ball-bouncing task and identified active modulations of the racket trajectory. However, several essential aspects were different in this previous study. First, the perturbations were applied in terms of changes in the coefficient of restitution α , a parameter of the model, and not in terms of ball release velocity, a state variable of the model. Therefore the actual perturbation effects on the height error depended also on the ball and racket velocity. As perturbation magnitudes were not as accurately controlled as in the present study, the resulting effects were not analyzed as a function of perturbation magnitude but rather pooled over all perturbations. Further, small perturbations were excluded to ensure that effects were observable. As no analyses of the basin of attraction were available, the effect of the stability boundary on behavior could not be addressed. Also the virtual set-up only provided a visual interface and was therefore not as realistic as the current development with the haptic contact. Despite these differences, the findings clearly indicated that actors adjusted their racket movement periods not amplitudes to re-establish the stable pattern, i.e., they actively tracked passive stability (see also Dijkstra et al. 2004) (Fig. 11). In contrast, the present analyses revealed significant modulations of both racket period and amplitude. This difference in results can be explained by noticing that the perturbations of de Rugy et al. were on average 0.45 m/s in absolute value with a range from 0.3 to 0.6 m/s (the changes were designed as random changes in α values which can be converted into velocities assuming an average ball velocity). This roughly corresponds to the perturbations $P-3$ to $P+3$. In Figs. 11 and 12, it can be observed that the modulations in racket period were indeed relatively larger than the modulations in amplitude for that range of perturbations with $\alpha = 0.5$. Thus we believe the difference in results can be explained by the relatively small range of perturbations in the de Rugy et al. study.

The present work built on this experiment but significantly developed the theoretical framework and fine-tuned the experimental approach to afford the testing of model-based quantitative and qualitative predictions about the effects of perturbations on dynamically stable behavior.

Overall the relaxation behavior was considerably faster than the model predicted. Based on the assumption that the racket trajectory remains unchanged in the face of perturbations, the model predicted that the return to steady state should take two to several tens of bounces for perturbations, even when the perturbation was still inside the basin of attraction. Solutions other than period-1 or the sticky solutions as predicted for perturbations outside the basin of attraction were never observed. This shows that in all cases, participants actively

accelerated their returns to the preperturbation steady state to as fast as one to three bounces. This fast return behavior was accomplished by active modulation of the racket trajectory in both amplitude and period. Analysis of the continuous trajectory showed deviations in the first cycle after the perturbation. Interestingly, the results of the exit times provided no signs of sensitivity to the boundary of the basin of attraction (prediction 1). Both the performance measure height error and also the racket trajectories showed gradually more pronounced adaptations to increasing perturbation magnitudes as stated in prediction 3. Support for this gradual change in racket kinematics was seen in the analysis of exit times, i.e., when the racket trajectory deviated from the typical trajectory in unperturbed conditions (Figs. 8 and 9). This indicated that for all applied errors some compensatory behavior is seen despite the dynamic stability afforded by the task. The only caveat to this conclusion is presented by the smallest perturbations as the return back to steady state was very fast and did not lead to significant deviations of the racket trajectory.

Despite this clear indication of active error compensation, the focal-dependent measure, acceleration of the racket at contact, exhibited very little changes for all but the three largest negative perturbations. Consistent with previous studies this demonstrated that conditions for passive stability, negative acceleration at impact, were maintained or immediately reestablished. This permits the conclusion that the racket trajectory is modulated to optimize the next ball contact, which also implies that actors may set up conditions for passive stability to assist return to steady state.

Further support for the sensitivity to passive dynamic stability properties are provided by the results that showed *qualitative* agreement with the second set of predictions of the model. First, the return to steady state after a perturbation took longer for larger perturbation magnitudes in all α conditions. This was evidenced by the increasingly longer relaxation times of the ball height errors over successive cycles after the perturbation with larger perturbation magnitudes. Second, relaxation times were shorter for positive compared with negative perturbations of corresponding magnitudes, mirroring the asymmetry in the basin of attraction. Third, lower α conditions exhibited faster returns for positive perturbations than higher α conditions. This is consistent with the topology of the basin of attraction, which is wider for higher α conditions but only on the positive side.

For the three most negative perturbations ($P-7$, $P-6$, $P-5$) high positive impact accelerations were observed in the two cycles after the perturbation, coincident with relatively large changes in the racket amplitudes. A first conjecture to account for these parallel changes in acceleration and amplitude was that the changes in acceleration were merely a consequence of the increase in racket amplitude. To test this, we calculated the racket accelerations that would follow from the observed increase in amplitude assuming sinusoidal racket movement. The calculation used a sine wave with the average racket period (0.65 s) and the average amplitudes measured for the four α conditions. The resulting maximum accelerations were 2.33, 3.36, 4.67, and 6.53 m/s² for the four α conditions, respectively. These values are significantly lower than the values found at C1 for the largest negative perturbation (Fig. 10). Hence, the observed large positive accelerations reflect more than just a byproduct of the increase in amplitude. Rather

they are induced to compensate for the perturbation. Further support for this interpretation comes from the observation that the values of AC on bounce C1 scaled with α values (see 1st and 2nd panels of Fig. 10). The higher the α , the less forceful an impact was required, thus the AC values were lower.

The findings of the present study support the conclusion that in the ball-bouncing task, human actors adopt a strategy of blending active control with exploiting the passive dynamics of the task. Active control and passive stability have been studied extensively in human walking and walking robots. Given that the basin of attraction of the passive walkers is narrow and the passive dynamic walker is not very resistant to perturbations, many extensions of the original model have included active control for stabilization (Schwab and Wisse 2001; Wisse and van Frankenhuisen 2003). Specifically, lateral stability in 3D walkers requires additional control or constraints based on sensory information and feedback to come close to performance in biological walkers.

These considerations lead to the intriguing question about the role of perceptual information for achieving or tuning stable behavior. For walking, Bauby and Kuo (2000) suggested that humans harness passive dynamic properties of the limb in the sagittal plane, but they also include perceptually guided active control to stabilize lateral motion. In ball bouncing, little work has examined the role of perceptual information. In one comparison of performance with visual and haptic information, the results showed that both types of information provide sufficient information to maintain the bouncing action, even though haptic information ensure more stable behavior than visual information alone (Sternad et al. 2000). In a two-dimensional extension of the task, where the actor manipulated two planar paddles, one by each hand, to “juggle” a puck back and forth on a low-friction air-hockey table, Ronsse and colleagues showed that visual information alters the behavioral strategies that humans adopt compared with performance with haptic information only (Ronsse et al. 2006a,b). However, given that the dimensionality of this task is higher, it is also likely that perceptual information plays a more important role.

An active form of control has been implemented by Bühler, Koditschek, and colleagues (Bühler and Koditschek 1990; Bühler et al. 1994). In their studies on ball bouncing, using a planar robot arm that bounces a puck in 2D, or a ball in 3D, they implemented the so-called mirror law. With this algorithm, the racket movements “mirror” the down coming ball movements such that the velocity of the racket is tightly coupled to the visually perceived ball velocity. One interesting outcome of this tight perception-action coupling is that the ball-racket contact then occurs at positive impact accelerations, which, according to our model, produces unstable solutions. Nevertheless, using this mirror algorithm, the robot juggler obtained successful bouncing actions in 2D and 3D. This may be one possible explanation for the findings in novice subjects who tend to use positive accelerations at contact (Dijkstra et al. 2004; Morice et al. 2007; Sternad et al. 2000).

In summary, the present study on perturbed ball bouncing revealed that actors apply a blend of passive stabilization and active perceptual control. On the one hand, the actor utilizes the passive stability afforded by the task and the performance qualitatively bears the signature of the basin of attraction derived from a mechanical model of the task. On the other hand, there were significant modulations of the actions on

changes brought about by externally applied perturbations of all sizes. The theoretically predicted boundary of stability did not seem to induce significant qualitative changes in performance. These observations are consistent with the notion that actors stabilize and flexibly control their behavior in the context of task, environmental, and informational constraints (e.g., Newell 1986; Warren 2006). The task affords passive stability, perturbations may arise from environmental influences, and errors introduced by external perturbations provide information to guide the adaptive corrections. To account for this blend of control, perceptually based tuning of the passively stable dynamics should be included in the purely passive ball bouncing model. This will be left for future work.

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GRANTS

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